Engineering Notes

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H 80-009

Impact of a Rigid Plate on a Half-Strip of Incompressible Liquid

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SUPPOSE the half-strip DOHD of width h (Fig. 1) of an incompressible liquid of density ρ and the rigid walls DO and OH_∞ are at rest before the moment of time $t=0$. At the moment $t=0$ the side wall OH_∞ is impulsively forced to move in the positive direction on the x-axis and it continues to move during the period of time $0<t\leq\Delta t$. Let the velocity of the wall OH_∞ at $t=\Delta t$ be u_0 and let the pressure p at the free surface HD be equal to zero.

To obtain the results for velocity distributions v_x and v_y within the liquid at $t=\Delta t$, it is necessary to solve the boundary value problem[1] for the Laplace equation $\nabla^2\phi=0$. (ϕ is the velocity potential $v_x=\partial\phi/\partial x$ and $v_y=\partial\phi/\partial y$) with the boundary conditions $v_y(\mathrm{OD})=0$, $v_x(\mathrm{OH})=u_0$, $\phi(\mathrm{HD})=0$, and $\phi(\mathrm{D})=0$.

For the Fourier cosine transformation[2]

$$\bar{\phi}_c=\sqrt{2/\pi}\int_0^\infty \phi\cdot\cos(\lambda x)\,\mathrm{d}x$$

we have

$$\bar{\phi}_c=-\sqrt{2/\pi}\,(u_0/\lambda^2)\cdot\{1-[\cosh(\lambda y)/\cosh(\lambda h)]\}$$

and then

$$(X=\pi x/2h,\qquad X^*=\pi x^*/2h,\qquad Y=\pi y/2h)$$

$$v_y=\sqrt{2/\pi}\cdot\int_0^\infty(\partial\bar{\phi}_c/\partial y)\cos(\lambda x)\,\mathrm{d}\lambda=\frac{u_0}{\pi}\ln\frac{\cosh X+\sin Y}{\cosh X-\sin Y}$$

On free surface

$$v_y(\mathrm{HD})=(2u_0/\pi)\cdot\ln\coth(\pi x/4h)$$

Now

$$v_x=\int_h^y\frac{\partial v_y}{\partial x}\cdot\mathrm{d}y=\frac{2u_0}{\pi}\cdot\arctan\left(\frac{\cos Y}{\sinh X}\right)$$

The velocity $v_x(x>0, y=h)=0$, and the point H is the point of discontinuity for v_x!

At the wall OH

$$\phi(0,y)=\int_{+\infty}^0 v_x\mathrm{d}x=-(4u_0h/\pi^2)\,\mathrm{D}(Y)$$

$$\mathrm{D}(Y)=\int_0^{+\infty}\arctan\left(\frac{\cos Y}{\sinh X^*}\right)\mathrm{d}X^*$$

$$=2\cos(Y)\cdot\int_0^\infty\frac{X^*\cdot\cosh X^*\cdot\mathrm{d}X^*}{\cosh(2X^*)-\cos(2Y+\pi)}$$

$$=\pi\cdot\ln(2)-2\cdot\{L(\pi/4+Y/2)+L(\pi/4-Y/2)\}$$

where L is Lobachevskiy's function.[3]

At the point O the velocity potential $\phi(\mathrm{O})=-(8u_0h/\pi^2)G$, G is Catalan's constant,[3] $G=0.91596\ldots$ In general

$$\phi=-\frac{4u_0h}{\pi^2}\cdot\mathrm{D}(Y)\cdot\left\{1-\frac{1}{\mathrm{D}(Y)}\int_0^{\pi x/2h}\arctan\left(\frac{\cos Y}{\sinh X^*}\right)\mathrm{d}X^*\right\}$$

For the total impulse P of pressure forces on the side wall OH_∞ for the period of time $0\leq t\leq\Delta t$ and for the unit of the length perpendicular to the plane of Fig. 1 we have

$$P(OH_\infty)=-\rho\int_0^h\phi(o,y)\,\mathrm{d}y=\frac{8u_0h^2\rho}{\pi^3}\cdot\int_{Y=0}^{Y=\pi/2}\mathrm{D}(Y)\,\mathrm{d}Y$$

$$\int_0^{\pi/2}\mathrm{D}(Y)\,\mathrm{d}Y=(\pi^2/2)\ln(2)-2(I_1+I_2)$$

$$I_1=\int_0^{\pi/2}L\left(\frac{\pi}{4}+\frac{Y}{2}\right)\mathrm{d}Y;\quad I_2=\int_0^{\pi/2}L\left(\frac{\pi}{4}-\frac{Y}{2}\right)\mathrm{d}Y$$

$$I_1=-\int_{Y=0}^{Y=\pi/2}\mathrm{d}Y\int_{\epsilon=0}^{\epsilon=\pi/4}\ln\cos\epsilon\,\mathrm{d}\epsilon-\int_{\epsilon=\pi/4}^{\epsilon=\pi/2}\ln\cos\epsilon\,\mathrm{d}\epsilon\int_{Y=2\epsilon-(\pi/2)}^{Y=\pi/2}\mathrm{d}Y$$

$$I_2=-\int_{\epsilon=0}^{\epsilon=\pi/4}\ln\cos\epsilon\,\mathrm{d}\epsilon\int_{Y=0}^{Y=(\pi/2)-2\epsilon}\mathrm{d}Y$$

$$I_1+I_2=\int_0^{\pi/2}(2\epsilon-\pi)\ln\cos\epsilon\,\mathrm{d}\epsilon=-2\int_0^{\pi/2}\epsilon\ln\sin\epsilon\,\mathrm{d}\epsilon$$

$$P(OH_\infty)=\frac{8u_0h^2\rho}{\pi^3}\cdot\left\{\frac{\pi^2\ln 2}{2}+4\cdot\int_0^{\pi/2}\epsilon\ln\sin\epsilon\,\mathrm{d}\epsilon\right\}$$

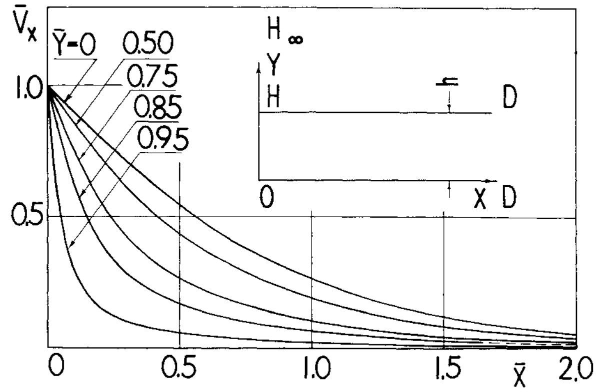

Fig. 1 Dimensionless horizontal velocity.

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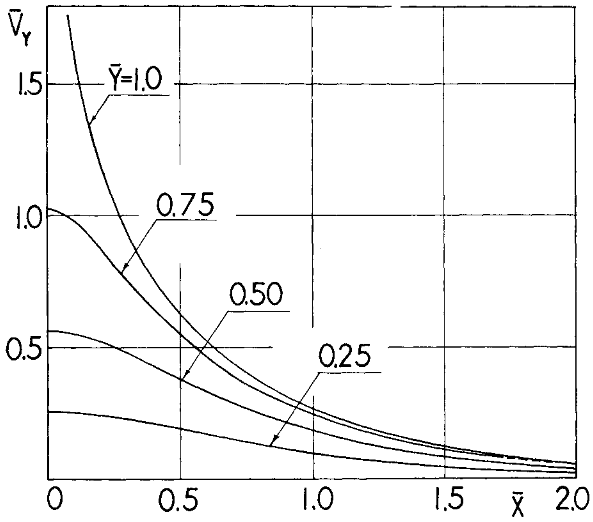

Fig. 2 Dimensionless vertical velocity.

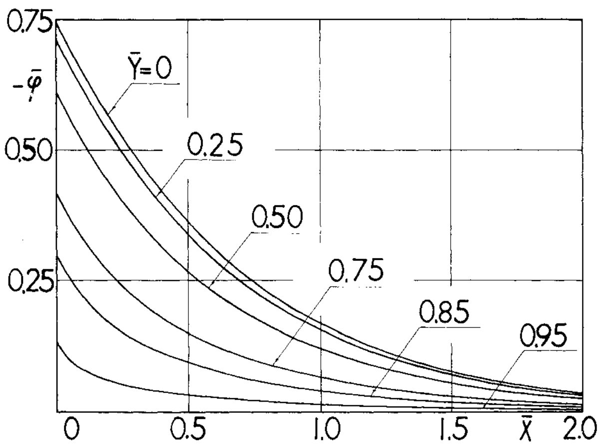

Fig. 3 Dimensionless velocity potential.

In the same manner

$$P(\mathrm{OD}) = -\rho \cdot \int_0^\infty \phi(x,0)\,\mathrm{d}x$$

$$= \frac{4u_0 h\rho}{\pi^2} \cdot \int_{x=0}^{x=\infty} \mathrm{d}x \int_X^\infty \arctan \frac{1}{\sinh X^*}\,\mathrm{d}X^*$$

$$= \frac{4u_0 h\rho}{\pi^2} \int_{X^*=0}^{X^*=\infty} \arctan\left(\frac{1}{\sinh X^*}\right) \mathrm{d}X^* \int_0^{x^*} \mathrm{d}x = (1/2)\rho u_0 h^2$$

The curves

$$\bar{v}_x = v_x/u_0, \qquad \bar{v}_y = v_y/u_0,$$

and

$$-\bar{\phi} = -\phi/(u_0 h) = (\rho u_0 h)^{-1} \cdot \int_0^{\Delta t} p\,\mathrm{d}t$$

as functions of $\bar{x} = x/h$ for some $\bar{y} = y/h$ are given in Figs. 1-3, respectively. Two specific results are: $\bar{\phi}(\mathrm{O}) = -0.7425$ and $P(\mathrm{OH})/(\rho u_0 h^2) = 0.5428$.

The influence of impact is mainly observed by the part of the liquid half-strip within the range $0 \leq \bar{x} \leq 2$.

Acknowledgment

This work was performed while the author was with Payne, Inc.

References

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[2] Lebedev, N.N., Skalskaya, I.P., and Ufluand Y.S., *Worked Problems in Applied Mathematics*, Dover Publications, New York, 1979.

[3] Gradshteyn, I.S. and Ryzhick, I.M., *Table of Integrals, Series and Products*, Academic Press, New York, 1965.